Do all the problems without calculators. All answers must be exact.

**Part I.**

1. Find : .

2. Find  for  at the point (3,–2).

 

At (3,–2): 

3. Show that for the equation ,

.



 Q.E.D.

4. . Find  when .

 

 When :

 

5. . Find .

 

6. The equation of a horizontal ellipse about the origin can be represented by the set of parametric equations: , , where *a* and *b* are positive constants.

a. Find :



 

b. Suppose *a* = 4 and *b* = –6. At what point(s) will the slope of a tangent = 45°?

This poorly worded question is attempting to ask where the slope will actually be equal to 1?

Thus, the points would be:



Using unit circles or dilated circles, these simplify to:



c. The radius of an ellipse is any segment from the center of the ellipse to a point on the ellipse. At what point will the radius of the ellipse be perpendicular to the tangent to the ellipse at that point?

The slope from the center (0,0) to any point on the ellipse: 

 To be perpendicular, the slope from the center to the point must be the negative reciprocal of the slope of the tangent. In other words:

 

This has six solutions:

 

 This means that if *a* and *b* are equal (making the shape a circle), then everywhere on the circle will be perpendicular to its radius. Otherwise, there are these solutions:

**Part II.**

7. Find .



8. Find .

 

9. Find an equation for the tangent to the curve at the indicated point.

 

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At , 

 

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10. A parabola is implicitly defined by the equation 

a. Find the point(s) at which a tangent to the slope would be vertical.

The tangent to the slope would be vertical at any point where its slope is undefined-infinite. That occurs when the denominator is zero and the numerator is not. In other words, when 

 

This would be vertical when

 

Substitute *x* for *y* in the original equation:

 

Thus, the slope of the tangent would be vertical at (–2,–2).

b. Find the point(s) at which the tangent to the slope would be horizontal.

The tangent would be horizontal when its slope if equal to zero:

 

Substitute into the original equation:

 

Thus the slope would be horizontal at the point (–1,–3)