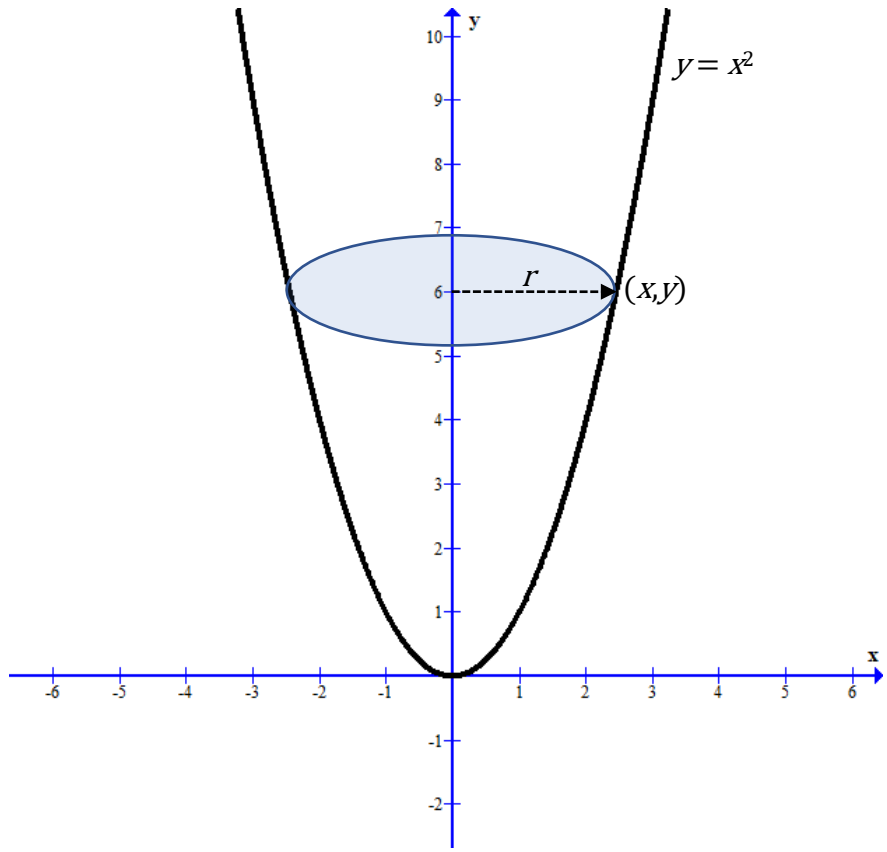
Look at the shape on the right. It represents the graph of a parabola that has been "rotated about the *y*-axis". If you could see it in three dimensions, it would look like the nose cone of a rocket facing down.

Next, look carefully at the *cross-section* illustrated in the picture. That shape is a **circle**. The circle is inscribed in the rotated parabola. It has a radius labelled *r*. In the picture, the inscribed circle was placed at the height of $y = 6$. But the inscribed circle could be placed at any height.

What is the circumference of the circle? The circumference is $2\pi r$.

What is the area of the circle? The area is πr^2.



Express the circumference in terms of *x*: Since the radius also represents the distance to the graph, we can replace *r* with *x*. In this picture, the circumference would be $2\pi x$. What would the area be? The answer would be: The area of the inscribed circle is πx^2.

Let's try to express the circumference in terms of *y*. We know that $y = x^2$. Solve for *x*, keeping in mind we only want the positive value because it represents a positive distance.

$$y = x^2$$
$$x^2 = y$$
$$x = \sqrt{y}$$

So the circumference of the circle is $2\pi\sqrt{y}$.

What about the area. Express the area in terms of *y*. You should get that the area is $\pi r^2 = \pi\left(\sqrt{y}\right)^2 = \pi y$.

This is the beginning of practice visualizing shapes that have been rotated and expressing information about them in terms of some assigned variable.

Let's start with some basics.

Write a general equation for the horizontal distance (r) from the y-axis to the graph of each of the following functions:

(1) $y = 2x, \quad x \le 0$

(2) $y = -x^2 + 9, \quad x \ge 0$

(3) $y = \sqrt{x^2 - 36}, \quad x \ge 6$

(4) $y = \sqrt{49 - x^2}, \quad 0 \le x \le 7$

(5) $y = \ln(x), \quad \frac{1}{e} \le x \le e$

If the graphs of each of the above functions were to be rotated about its y-axis, it would result in a 3D object with circular cross-sections – <u>just as in the example on the previous page</u>. Use the example to help you answer the following questions.

For problems 6 – 10, you will need to use the equations in 1 – 5, respectively, to find the general equation for the area in terms of y of a cross-section of the rotated graph of the given equation.

For each problem set up an integral to find the sum of the areas of all the cross-sections of the rotated graph.

Try to evaluate the integral. If you find you need a calculator, go ahead and use one.