

Essential Algebra Skills – Strategies For Solving Linear Equations

The goal of solving for a linear variable in an equation is to isolate that variable on one side or the other of an equation.

The following are all “legal” steps in the process.

1. You may always simplify any expression in the equation.
2. You may “flip” or “reverse” the equation.
3. You may change the sign of every term in the equation.
4. You may add or subtract the same number or expression to or from both sides of the equation.
5. You may multiply or divide both sides of the equation by any number or expression that is not equal to 0.

Examples:

Equation	Step to do next
$2x + 3x - 2 = x - 8 + 5$	Simplify (combine like terms)
$5x - 2 = x - 3$	

Equation	Step to do next
$x + 6 = 7x - 12$	Reverse the equation
$7x - 12 = x + 6$	

Equation	Step to do next
$x + 6 = 7x - 12$	Change the signs
$-x - 6 = -7x + 12$	

Equation	Step to do next
$x + 6 = 7x - 12$	Add 12 to each side of the equation
$x + 18 = 7x$	Subtract x from each side of the equation
$18 = 6x$	Divide every term by 6
$3 = x$	

Equation	Step to do next
$\frac{x}{5} = 7$	Multiply both sides of the equation by 5
$x = 35$	

There is an easier way to think about your steps. Instead of thinking about adding, subtracting, multiplying, or dividing a number from each side, think of the step as removing a term or a coefficient.

1. You can remove a term that is added or subtracted to one side of an equation by doing the opposite operation with it to the other side.
2. You can remove a coefficient from any term by dividing all the other terms by the same number.
3. You can remove a divisor from any term by multiplying all the other terms by the same number.

Example:

Equation	Step to do next
$2x + 3x - 2 = x - 8 + 5$	Simplify (combine like terms)
$5x - 2 = x - 3$	Remove the “subtract 2” from the left side by “adding 2” to the right side.
$5x = x - 3 + 2$	Simplify (combine like terms)
$5x = x - 1$	Remove the “plus x ” from the right side by “subtracting x ” from the left side.
$5x - x = -1$	Simplify (combine like terms)
$4x = -1$	Remove the coefficient 4 by dividing the other terms by 4.
$x = -\frac{1}{4}$	

Once you understand this, you can do any of the simplifying steps in your head or on scratch paper.

Example:

Equation	Step to do next
$2x + 3x - 2 = x - 8 + 5$	Simplify (combine like terms)
$5x - 2 = x - 3$	Remove the “subtract 2” from the left side by “adding 2” to the right side.
$5x = x - 1$	Remove the “plus x ” from the right side by “subtracting x ” from the left side.
$4x = -1$	Remove the coefficient 4 by dividing the other terms by 4.
$x = -\frac{1}{4}$	

Last you may also combine the adding/subtracting steps into a single step.

Example:

Equation	Step to do next
$2x + 3x - 2 = x - 8 + 5$	Simplify (combine like terms)
$5x - 2 = x - 3$	Remove the “subtract 2” from the left side by “adding 2” to the right side. Remove the “plus x ” from the right side by “subtracting x ” from the left side.
$4x = -1$	Remove the coefficient 4 by dividing the other terms by 4.
$x = -\frac{1}{4}$	

Here are examples of how to think about and solve a linear equation:

Equation	Goal	Think	Scratch Work
$7x + 2 = 3x - 15$	Remove the $3x$	Subtract $3x$ from both sides	$7x - 3x = 2x$
$2x + 2 = -15$	Remove the “add 2”	Subtract 2 from both sides	$-15 - 2 = -17$
$2x = -17$	Remove the coefficient 2	Divide both sides by 2	$\frac{-17}{2} = -\frac{17}{2} = -8\frac{1}{2}$
$x = -8\frac{1}{2}$			

Equation	Goal	Think	Scratch Work
$-5x + 3 = 2x - 11$	Change the sign of the first term	Change the sign of every term	
$5x - 3 = -2x + 11$	Remove the “subtract 3”	Add 3 to both sides	$11 + 3 = 14$
$5x = -2x + 14$	Remove the “minus $2x$ ”	Add $2x$ to both sides	$5x + 2x = 7x$
$7x = 14$	Remove the coefficient 7	Divide both sides by 7	$\frac{14}{7} = 2$
$x = 2$			

You can make a “table of steps” to show others what you are doing.

Example:

Equation	Step	(Means)
Solve for y $3x - 8y = 29$	$-3x$	(Subtract $3x$ from both sides)
$-8y = -3x + 29$	$\frac{\quad}{-8}$	Divide all terms by -8
$y = \frac{-3x}{-8} + \frac{29}{-8}$	Simplify	simplify
$y = \frac{3}{8}x - 3\frac{5}{8}$		